All of the other Exponential and Logarithmic Models.

All of the models we have used have been of the form $y = ab^x$ (basic, half-life, % increase, interest), but there are many more, and many have very different forms.

Newton's Law of Cooling: When a hot object is placed in a cooler place, it will slowly cool off until it reaches the temperature of the space it is put in.

The equation that models this is called Newton's Law of Cooling: $T = T_m + (T_o - T_m)e^{-kt}$

Where $T_m = temperature\ of\ the\ medium\ (what\ it\ cools\ in)$ $T_o = initial\ temperature$

k = cooling constant and T = temperature at time = t.

Ex: You bake a cake at 350° F. You place the cake in a room that is 30° F. In 5 minutes, the cake has cooled down to 200° F.

A. What will be the Temperature of the cake in 10 minutes?

First, find k. $200 = 30 + (350 - 30)e^{-k(5)} \rightarrow 170 = (320)e^{-5k} \rightarrow 0.53125 = e^{-5k}$

Convert: $-5k = \ln(0.53125) = -0.6325 \rightarrow k = 0.1265$

Second, solve the problem we want $T = 30 + (350 - 30)e^{-0.1265(10)} \rightarrow T = 30 + 320e^{-1.265} \rightarrow$

$T = 30 + (320)(0.2822) \rightarrow T = 30 + 90.317 = 120.32^oF$

B. How long until the cake is 60° F?

$60 = 30 + (320)e^{-0,1265t} \rightarrow 30 = (320)e^{-0.1265t} \rightarrow 0.09375 = e^{-0.1265t}$

Convert: $-0.1265t = \ln(0.09375) = -2.367 \rightarrow t = 18.712\ minutes$

Logistic Growth: The basic logistic function is $f(x) = \frac{L}{1+e^{-k(x-x_0)}}$.

This model is used for lots of applications from population growth, to the growth of tumors, to the spread of rumors. In these models, there is an initial amount of growth that eventually slows down to nothing.

Ex: The spread of the flu in a high school can be modeled by the equation: $y = \frac{1000}{1+999e^{-0.8t}}$.

y = people with the flu t = time (in this case) days

A. How many people are initially infected? $t = 0$: $y = \frac{1000}{1+999e^{-0.8(0)}} = \frac{1000}{1+999} = \frac{1000}{1000} = 1$

B. What is the maximum number of people who will be infected? $t = \infty$: $y = \frac{1000}{1+0} = 1000$

C. How long until half of the students are infected? $500 = \frac{1000}{1+999e^{-0.8(0)}} \rightarrow 500(1 + 999e^{-0.8t}) = 1000$

$1 + 999e^{-0.8t} = 2 \rightarrow 999e^{-0.8t} = 1 \rightarrow e^{-0.8t} = 0.001 \rightarrow -0.8t = \ln(0.001) = -6.907 \rightarrow t = 8.63\ days$

Product Demand: The demand for a certain product is given by the equation: $p = 500 - 0.5e^{0.004x}$, where p is price and x is demand. What would be the demand for the product if the price was $300?

$300 = 500 - 0.5e^{0.004x} \rightarrow -200 = -0.5e^{0.004x} \rightarrow 400 = e^{0.004x}$

$0.004x = \ln(400) = 5.99 \rightarrow x = 1497.8$